

PDE Provider

Course Title

Display Data

Instructor

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Credit

3 PDU

Questions

15

Adaptation Statement

- *This course is adapted from chapter 2 titled “Descriptive Statistics” from the book titled “Introductory Business Statistics 2e”, which can be downloaded for free from the following links:*

<https://open.umn.edu/opentextbooks/textbooks/introductory-business-statistics-2017>

- *The book “Introductory Business Statistics 2e” is used under a Creative Commons Attribution License, except where otherwise noted.*



- *Check additional references and sources in the original document.*
- *This adaptation has reformatted the original text, and have replaced some images and figures to make the resulting whole more shareable.*

2.1 Display Data

Stem-and-Leaf Graphs (Stemplots), Line Graphs, and Bar Graphs

One simple graph, the **stem-and-leaf graph** or **stemplot**, comes from the field of exploratory data analysis. It is a good choice when the data sets are small. To create the plot, divide each observation of data into a stem and a leaf. The leaf consists of a **final significant digit**. For example, 23 has stem two and leaf three. The number 432 has stem 43 and leaf two. Likewise, the number 5,432 has stem 543 and leaf two. The decimal 9.3 has stem nine and leaf three. Write the stems in a vertical line from smallest to largest. Draw a vertical line to the right of the stems. Then write the leaves in increasing order next to their corresponding stem.

EXAMPLE 2.1

For Professor Dean's spring pre-calculus class, scores for the first exam were as follows (smallest to largest): 33; 42; 49; 49; 53; 55; 55; 61; 63; 67; 68; 68; 69; 69; 72; 73; 74; 78; 80; 83; 88; 88; 88; 88; 90; 92; 94; 94; 94; 94; 96; 100

| Stem | Leaf |
|------|---------------|
| 3 | 3 |
| 4 | 2 9 9 |
| 5 | 3 5 5 |
| 6 | 1 3 7 8 8 9 9 |
| 7 | 2 3 4 8 |
| 8 | 0 3 8 8 8 |
| 9 | 0 2 4 4 4 4 6 |
| 10 | 0 |

Table 2.1 Stem-and-Leaf Graph

The stemplot shows that most scores fell in the 60s, 70s, 80s, and 90s. Eight out of the 31 scores or approximately 26% ($\frac{8}{31}$) were in the 90s or 100, a fairly high number of As.

TRY IT 2.1

For the Park City basketball team, scores for the last 30 games were as follows (smallest to largest): 32; 32; 33; 34; 38; 40; 42; 42; 43; 44; 46; 47; 47; 48; 48; 48; 49; 50; 50; 51; 52; 52; 52; 53; 54; 56; 57; 57; 60; 61
Construct a stem plot for the data.

The stemplot is a quick way to graph data and gives an exact picture of the data. You want to look for an overall pattern and any outliers. An **outlier** is an observation of data that does not fit the rest of the data. It is sometimes called an **extreme value**. When you graph an outlier, it will appear not to fit the pattern of the graph. Some outliers are due to mistakes (for example, writing down 50 instead of 500) while others may indicate that something unusual is happening. It takes some background information to explain outliers, so we will cover them in more detail later.

EXAMPLE 2.2

The data are the distances (in kilometers) from a home to local supermarkets. Create a stemplot using the data:
 1.1; 1.5; 2.3; 2.5; 2.7; 3.2; 3.3; 3.3; 3.5; 3.8; 4.0; 4.2; 4.5; 4.5; 4.7; 4.8; 5.5; 5.6; 6.5; 6.7; 12.3

🔍 **Problem**

Do the data seem to have any concentration of values?

NOTE

The leaves are to the right of the decimal.

✅ **Solution**

The value 12.3 may be an outlier. Values appear to concentrate at three and four kilometers.

| Stem | Leaf |
|------|-------------|
| 1 | 1 5 |
| 2 | 3 5 7 |
| 3 | 2 3 3 5 8 |
| 4 | 0 2 5 5 7 8 |
| 5 | 5 6 |
| 6 | 5 7 |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | 3 |

Table 2.2

> **TRY IT 2.2**

The following data show the distances (in miles) from the homes of off-campus statistics students to the college. Create a stem plot using the data and identify any outliers:

0.5; 0.7; 1.1; 1.2; 1.2; 1.3; 1.3; 1.5; 1.5; 1.7; 1.7; 1.8; 1.9; 2.0; 2.2; 2.5; 2.6; 2.8; 2.8; 2.8; 3.5; 3.8; 4.4; 4.8; 4.9; 5.2; 5.5; 5.7; 5.8; 8.0

EXAMPLE 2.3**Problem**

A **side-by-side stem-and-leaf plot** allows a comparison of the two data sets in two columns. In a side-by-side stem-and-leaf plot, two sets of leaves share the same stem. The leaves are to the left and the right of the stems. [Table 2.3](#) and [Table 2.4](#) show the ages of presidents at their inauguration and at their death. Construct a side-by-side stem-and-leaf plot using this data.

| President | Age | President | Age | President | Age |
|----------------|-----|--------------|-----|--------------|-----|
| Washington | 57 | Lincoln | 52 | Hoover | 54 |
| J. Adams | 61 | A. Johnson | 56 | F. Roosevelt | 51 |
| Jefferson | 57 | Grant | 46 | Truman | 60 |
| Madison | 57 | Hayes | 54 | Eisenhower | 62 |
| Monroe | 58 | Garfield | 49 | Kennedy | 43 |
| J. Q. Adams | 57 | Arthur | 51 | L. Johnson | 55 |
| Jackson | 61 | Cleveland | 47 | Nixon | 56 |
| Van Buren | 54 | B. Harrison | 55 | Ford | 61 |
| W. H. Harrison | 68 | Cleveland | 55 | Carter | 52 |
| Tyler | 51 | McKinley | 54 | Reagan | 69 |
| Polk | 49 | T. Roosevelt | 42 | G.H.W. Bush | 64 |
| Taylor | 64 | Taft | 51 | Clinton | 47 |
| Fillmore | 50 | Wilson | 56 | G. W. Bush | 54 |
| Pierce | 48 | Harding | 55 | Obama | 47 |
| Buchanan | 65 | Coolidge | 51 | | |

Table 2.3 Presidential Ages at Inauguration

| President | Age | President | Age | President | Age |
|------------|-----|------------|-----|--------------|-----|
| Washington | 67 | Lincoln | 56 | Hoover | 90 |
| J. Adams | 90 | A. Johnson | 66 | F. Roosevelt | 63 |
| Jefferson | 83 | Grant | 63 | Truman | 88 |
| Madison | 85 | Hayes | 70 | Eisenhower | 78 |

Table 2.4 Presidential Age at Death

| President | Age | President | Age | President | Age |
|----------------|-----|--------------|-----|------------|-----|
| Monroe | 73 | Garfield | 49 | Kennedy | 46 |
| J. Q. Adams | 80 | Arthur | 56 | L. Johnson | 64 |
| Jackson | 78 | Cleveland | 71 | Nixon | 81 |
| Van Buren | 79 | B. Harrison | 67 | Ford | 93 |
| W. H. Harrison | 68 | Cleveland | 71 | Reagan | 93 |
| Tyler | 71 | McKinley | 58 | | |
| Polk | 53 | T. Roosevelt | 60 | | |
| Taylor | 65 | Taft | 72 | | |
| Fillmore | 74 | Wilson | 67 | | |
| Pierce | 64 | Harding | 57 | | |
| Buchanan | 77 | Coolidge | 60 | | |

Table 2.4 Presidential Age at Death

✓ **Solution**

| Ages at Inauguration | | Ages at Death | |
|---|---|-------------------------|--|
| 9 9 8 7 7 7 6 3 2 | 4 | 6 9 | |
| 8 7 7 7 7 6 6 6 5 5 5 5 4 4 4 4 4 2 2 1 1 1 1 1 1 0 | 5 | 3 6 6 7 7 8 | |
| 9 8 5 4 4 2 1 1 1 0 | 6 | 0 0 3 3 4 4 5 6 7 7 7 8 | |
| | 7 | 0 1 1 1 2 3 4 7 8 8 9 | |
| | 8 | 0 1 3 5 8 | |
| | 9 | 0 0 3 3 | |

Table 2.5

> **TRY IT 2.3**

The table shows the number of wins and losses the Atlanta Hawks have had in 42 seasons. Create a side-by-side stem-and-leaf plot of these wins and losses.

| Losses | Wins | Season | Losses | Wins | Season |
|--------|------|--------|--------|------|--------|
| 34 | 48 | 1 | 41 | 41 | 22 |
| 34 | 48 | 2 | 39 | 43 | 23 |
| 46 | 36 | 3 | 44 | 38 | 24 |
| 46 | 36 | 4 | 39 | 43 | 25 |
| 36 | 46 | 5 | 25 | 57 | 26 |
| 47 | 35 | 6 | 40 | 42 | 27 |
| 51 | 31 | 7 | 36 | 46 | 28 |
| 53 | 29 | 8 | 26 | 56 | 29 |
| 51 | 31 | 9 | 32 | 50 | 30 |
| 41 | 41 | 10 | 19 | 31 | 31 |
| 36 | 46 | 11 | 54 | 28 | 32 |
| 32 | 50 | 12 | 57 | 25 | 33 |
| 51 | 31 | 13 | 49 | 33 | 34 |
| 40 | 42 | 14 | 47 | 35 | 35 |
| 39 | 43 | 15 | 54 | 28 | 36 |
| 42 | 40 | 16 | 69 | 13 | 37 |
| 48 | 34 | 17 | 56 | 26 | 38 |
| 32 | 50 | 18 | 52 | 30 | 39 |
| 25 | 57 | 19 | 45 | 37 | 40 |
| 32 | 50 | 20 | 35 | 47 | 41 |
| 30 | 52 | 21 | 29 | 53 | 42 |

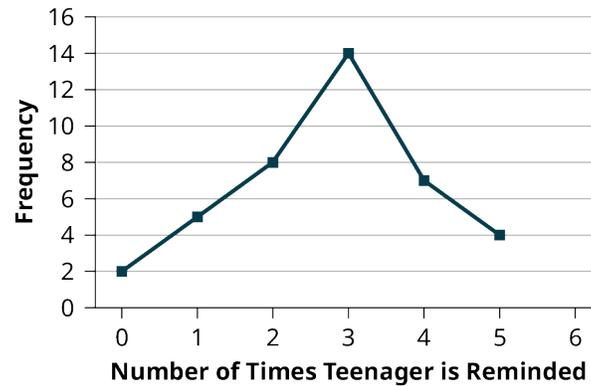
Table 2.6

Another type of graph that is useful for specific data values is a **line graph**. In the particular line graph shown in [Example 2.4](#), the **x-axis** (horizontal axis) consists of **data values** and the **y-axis** (vertical axis) consists of **frequency points**. The frequency points are connected using line segments.

EXAMPLE 2.4

In a survey, 40 parents were asked how many times per week a teenager must be reminded to do their chores. The results are shown in [Table 2.7](#) and in [Figure 2.2](#).

| Number of times teenager is reminded | Frequency |
|--------------------------------------|-----------|
| 0 | 2 |
| 1 | 5 |
| 2 | 8 |
| 3 | 14 |
| 4 | 7 |
| 5 | 4 |

Table 2.7**Figure 2.2****> TRY IT 2.4**

In a survey, 40 people were asked how many times per year they had their car in the shop for repairs. The results are shown in [Table 2.8](#). Construct a line graph.

| Number of times in shop | Frequency |
|-------------------------|-----------|
| 0 | 7 |
| 1 | 10 |
| 2 | 14 |
| 3 | 9 |

Table 2.8

Bar graphs consist of bars that are separated from each other. The bars can be rectangles or they can be rectangular boxes (used in three-dimensional plots), and they can be vertical or horizontal. The **bar graph** shown in [Example 2.5](#) has age groups represented on the **x-axis** and proportions on the **y-axis**.

EXAMPLE 2.5

Problem

The percentage of U.S.-based TikTok users by age is shown in [Table 2.9](#). Construct a bar graph using these data.

| Age groups | Proportion (%) of TikTok users |
|------------|--------------------------------|
| 10–19 | 32.5% |
| 20–29 | 29.5% |
| 30–39 | 16.4% |
| 40–49 | 13.9% |
| 50+ | 7.1% |

Table 2.9

Solution

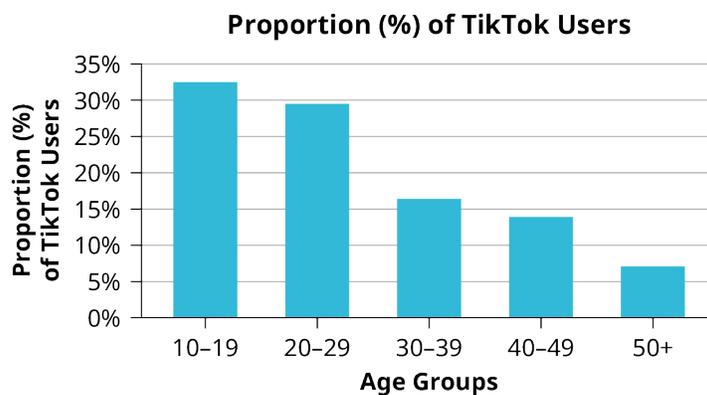


Figure 2.3

TRY IT 2.5

The population in Park City is made up of children, working-age adults, and retirees. [Table 2.10](#) shows the three age groups, the number of people in the town from each age group, and the proportion (%) of people in each age group. Construct a bar graph showing the proportions.

| Age groups | Number of people | Proportion of population |
|--------------------|------------------|--------------------------|
| Children | 67,059 | 19% |
| Working-age adults | 152,198 | 43% |
| Retirees | 131,662 | 38% |

Table 2.10

EXAMPLE 2.6

Problem

The columns in [Table 2.11](#) show the projected data for the year 2030 for the number and percentages of high school graduates by geographic region in the United States. Create a bar graph for these data with the geographic region (qualitative data) on the x-axis and the percentage of high school data (quantitative data) on the y-axis.

| Region | Number of Graduates | Percentage of Graduates |
|-----------|---------------------|-------------------------|
| Northeast | 517,720 | 16.1% |
| Midwest | 695,170 | 21.6% |
| South | 1,253,540 | 39.0% |
| West | 749,400 | 23.3% |

Table 2.11

Solution

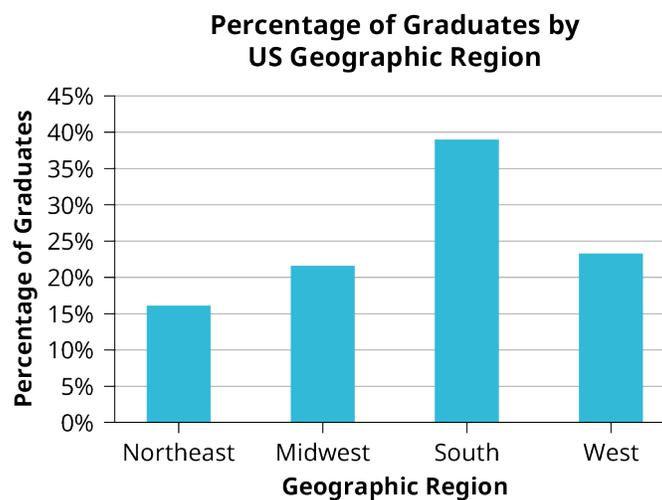


Figure 2.4

> TRY IT 2.6

Park City is broken down into six voting districts. The table shows the percent of the total registered voter population that lives in each district as well as the percent total of the entire population that lives in each district. Construct a bar graph that shows the registered voter population by district.

| District | Registered voter population | Overall city population |
|----------|-----------------------------|-------------------------|
| 1 | 15.5% | 19.4% |
| 2 | 12.2% | 15.6% |
| 3 | 9.8% | 9.0% |
| 4 | 17.4% | 18.5% |
| 5 | 22.8% | 20.7% |
| 6 | 22.3% | 16.8% |

Table 2.12

EXAMPLE 2.7

? Problem

Below is a two-way table showing the types of pets owned by men and women:

| | Dogs | Cats | Fish | Total |
|-------|------|------|------|-------|
| Men | 4 | 2 | 2 | 8 |
| Women | 4 | 6 | 2 | 12 |
| Total | 8 | 8 | 4 | 20 |

Table 2.13

Given these data, calculate the conditional distributions for the subpopulation of men who own each pet type.

✓ Solution

Men who own dogs = $4/8 = 0.5$

Men who own cats = $2/8 = 0.25$

Men who own fish = $2/8 = 0.25$

Note: The sum of all of the conditional distributions must equal one. In this case, $0.5 + 0.25 + 0.25 = 1$; therefore, the solution "checks".

> TRY IT 2.7

Given the data in [Table 2.13](#), calculate the conditional distributions for the subpopulation of women who own each

pet type.

Histograms, Frequency Polygons, and Time Series Graphs

For most of the work you do in this book, you will use a histogram to display the data. One advantage of a histogram is that it can readily display large data sets. A rule of thumb is to use a histogram when the data set consists of 100 values or more.

A **histogram** consists of contiguous (adjoining) boxes. It has both a horizontal axis and a vertical axis. The horizontal axis is labeled with what the data represents (for instance, distance from your home to school). The vertical axis is labeled either **frequency** or **relative frequency** (or percent frequency or probability). The graph will have the same shape with either label. The histogram (like the stemplot) can give you the shape of the data, the center, and the spread of the data.

The relative frequency is equal to the frequency for an observed value of the data divided by the total number of data values in the sample. (Remember, frequency is defined as the number of times an answer occurs.) If:

- f = frequency
- n = total number of data values (or the sum of the individual frequencies), and
- RF = relative frequency,

then:

$$RF = \frac{f}{n}$$

For example, if three students in an English class of 40 students received from 90% to 100%, then, $f = 3$, $n = 40$, and $RF = \frac{f}{n} = \frac{3}{40} = 0.075$. 7.5% of the students received 90–100%. 90–100% are quantitative measures.

To construct a histogram, first decide how many **bars** or **intervals**, also called classes, represent the data. Many histograms consist of five to 15 bars or classes for clarity. The number of bars needs to be chosen. Choose a starting point for the first interval to be less than the smallest data value. A **convenient starting point** is a lower value carried out to one more decimal place than the value with the most decimal places. For example, if the value with the most decimal places is 6.1 and this is the smallest value, a convenient starting point is 6.05 ($6.1 - 0.05 = 6.05$). We say that 6.05 has more precision. If the value with the most decimal places is 2.23 and the lowest value is 1.5, a convenient starting point is 1.495 ($1.5 - 0.005 = 1.495$). If the value with the most decimal places is 3.234 and the lowest value is 1.0, a convenient starting point is 0.9995 ($1.0 - 0.0005 = 0.9995$). If all the data happen to be integers and the smallest value is two, then a convenient starting point is 1.5 ($2 - 0.5 = 1.5$). Also, when the starting point and other boundaries are carried to one additional decimal place, no data value will fall on a boundary. The next two examples go into detail about how to construct a histogram using continuous data and how to create a histogram using discrete data.

EXAMPLE 2.8

The following data are the heights (in inches to the nearest half inch) of 100 male semiprofessional soccer players. The heights are **continuous** data, since height is measured.

60; 60.5; 61; 61; 61.5

63.5; 63.5; 63.5

64; 64; 64; 64; 64; 64; 64; 64; 64.5; 64.5; 64.5; 64.5; 64.5; 64.5; 64.5; 64.5

66; 66; 66; 66; 66; 66; 66; 66; 66; 66; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 67; 67; 67; 67; 67; 67; 67;

67; 67; 67; 67; 67; 67.5; 67.5; 67.5; 67.5; 67.5; 67.5; 67.5; 67.5

68; 68; 69; 69; 69; 69; 69; 69; 69; 69; 69; 69; 69.5; 69.5; 69.5; 69.5; 69.5

70; 70; 70; 70; 70; 70; 70.5; 70.5; 70.5; 71; 71; 71

72; 72; 72; 72.5; 72.5; 73; 73.5

74

The smallest data value is 60. Since the data with the most decimal places has one decimal (for instance, 61.5), we want our starting point to have two decimal places. Since the numbers 0.5, 0.05, 0.005, etc. are convenient numbers, use 0.05 and subtract it from 60, the smallest value, for the convenient starting point.

$60 - 0.05 = 59.95$ which is more precise than, say, 61.5 by one decimal place. The starting point is, then, 59.95.

The largest value is 74, so $74 + 0.05 = 74.05$ is the ending value.

Next, calculate the width of each bar or class interval. To calculate this width, subtract the starting point from the ending value and divide by the number of bars (you must choose the number of bars you desire). Suppose you choose eight bars.

$$74.05 - 59.95 = 14.1$$

$$14.1 \div 8 = 1.76$$

NOTE

We will round up to two and make each bar or class interval two units wide. Rounding up to two is one way to prevent a value from falling on a boundary. Rounding to the next number is often necessary even if it goes against the standard rules of rounding. For this example, using 1.76 as the width would also work. A guideline that is followed by some for the width of a bar or class interval is to take the square root of the number of data values and then round to the nearest whole number, if necessary. For example, if there are 150 values of data, take the square root of 150 and round to 12 bars or intervals.

The boundaries are:

- 59.95
- $59.95 + 2 = 61.95$
- $61.95 + 2 = 63.95$
- $63.95 + 2 = 65.95$
- $65.95 + 2 = 67.95$
- $67.95 + 2 = 69.95$
- $69.95 + 2 = 71.95$
- $71.95 + 2 = 73.95$
- $73.95 + 2 = 75.95$

The heights 60 through 61.5 inches are in the interval 59.95–61.95. The heights that are 63.5 are in the interval 61.95–63.95. The heights that are 64 through 64.5 are in the interval 63.95–65.95. The heights 66 through 67.5 are in the interval 65.95–67.95. The heights 68 through 69.5 are in the interval 67.95–69.95. The heights 70 through 71 are in the interval 69.95–71.95. The heights 72 through 73.5 are in the interval 71.95–73.95. The height 74 is in the interval 73.95–75.95.

The following histogram displays the heights on the x-axis and relative frequency on the y-axis.

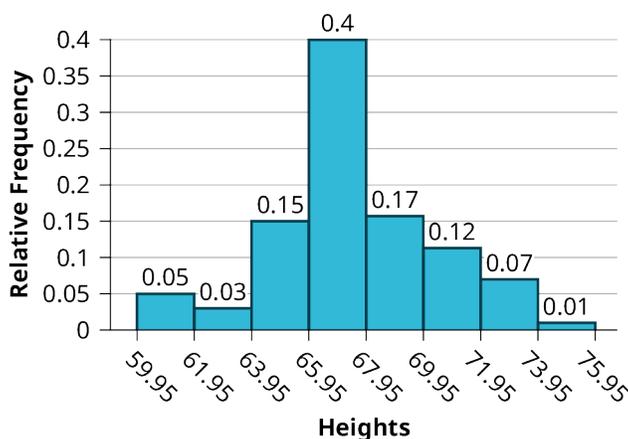


Figure 2.5

> TRY IT 2.8

The following data are the shoe sizes of 50 male students. The sizes are continuous data since shoe size is measured.

Construct a histogram and calculate the width of each bar or class interval. Suppose you choose six bars.
 9; 9; 9.5; 9.5; 10; 10; 10; 10; 10; 10.5; 10.5; 10.5; 10.5; 10.5; 10.5; 10.5
 11; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11.5; 11.5; 11.5; 11.5; 11.5; 11.5
 12; 12; 12; 12; 12; 12; 12.5; 12.5; 12.5; 12.5; 14

EXAMPLE 2.9

Create a histogram for the following data: the number of books bought by 50 part-time college students at ABC College. The number of books is **discrete data**, since books are counted.

- 1; 1; 1; 1; 1; 1; 1; 1; 1; 1
- 2; 2; 2; 2; 2; 2; 2; 2; 2
- 3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3
- 4; 4; 4; 4; 4
- 5; 5; 5; 5; 5
- 6; 6

Eleven students buy one book. Ten students buy two books. Sixteen students buy three books. Six students buy four books. Five students buy five books. Two students buy six books.

Because the data are integers, subtract 0.5 from 1, the smallest data value and add 0.5 to 6, the largest data value. Then the starting point is 0.5 and the ending value is 6.5.

Problem

Next, calculate the width of each bar or class interval. If the data are discrete and there are not too many different values, a width that places the data values in the middle of the bar or class interval is the most convenient. Since the data consist of the numbers 1, 2, 3, 4, 5, 6, and the starting point is 0.5, a width of one places the 1 in the middle of the interval from 0.5 to 1.5, the 2 in the middle of the interval from 1.5 to 2.5, the 3 in the middle of the interval from 2.5 to 3.5, the 4 in the middle of the interval from _____ to _____, the 5 in the middle of the interval from _____ to _____, and the _____ in the middle of the interval from _____ to _____.

Solution

- 3.5 to 4.5
- 4.5 to 5.5
- 6
- 5.5 to 6.5

Calculate the number of bars as follows:

$$6.5 - 0.5 = 6$$

$$6 \div 1 = 6$$

where 1 is the width of a bar. Therefore, bars = 6.

The following histogram displays the number of books on the x-axis and the frequency on the y-axis.

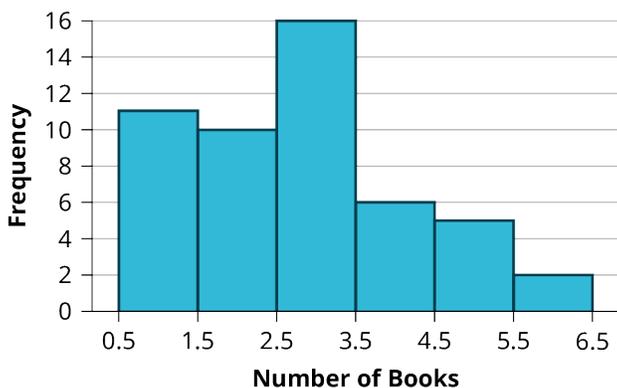


Figure 2.6

left boundary, but not if it falls on the right boundary. Different researchers may set up histograms for the same data in different ways. There is more than one correct way to set up a histogram.

> **TRY IT 2.10**

The following data represent the number of employees at various restaurants in New York City. Using this data, create a histogram.

22; 35; 15; 26; 40; 28; 18; 20; 25; 34; 39; 42; 24; 22; 19; 27; 22; 34; 40; 20; 38; and 28
Use 10–19 as the first interval.

 **COLLABORATIVE EXERCISE**

Count the money (bills and change) in your pocket or purse. Your instructor will record the amounts. As a class, construct a histogram displaying the data. Discuss how many intervals you think is appropriate. You may want to experiment with the number of intervals.

Frequency Polygons

Frequency polygons are analogous to line graphs, and just as line graphs make continuous data visually easy to interpret, so too do frequency polygons.

To construct a frequency polygon, first examine the data and decide on the number of intervals, or class intervals, to use on the x -axis and y -axis. After choosing the appropriate ranges, begin plotting the data points. After all the points are plotted, draw line segments to connect them.

EXAMPLE 2.11

A frequency polygon was constructed from the frequency table below.

| Frequency distribution for calculus final test scores | | | |
|---|-------------|-----------|----------------------|
| Lower bound | Upper bound | Frequency | Cumulative frequency |
| 49.5 | 59.5 | 5 | 5 |
| 59.5 | 69.5 | 10 | 15 |
| 69.5 | 79.5 | 30 | 45 |
| 79.5 | 89.5 | 40 | 85 |
| 89.5 | 99.5 | 15 | 100 |

Table 2.15

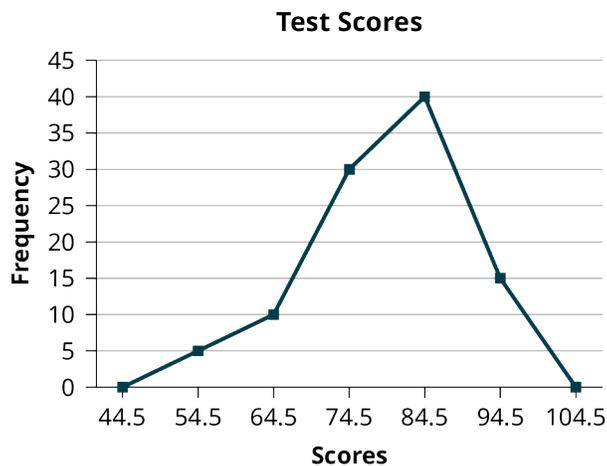


Figure 2.8

The first label on the x-axis is 44.5. This represents an interval extending from 39.5 to 49.5. Since the lowest test score is 54.5, this interval is used only to allow the graph to touch the x-axis. The point labeled 54.5 represents the next interval, or the first “real” interval from the table, and contains five scores. This reasoning is followed for each of the remaining intervals with the point 104.5 representing the interval from 99.5 to 109.5. Again, this interval contains no data and is only used so that the graph will touch the x-axis. Looking at the graph, we say that this distribution is skewed because one side of the graph does not mirror the other side.

> TRY IT 2.11

Construct a frequency polygon of U.S. Presidents’ ages at inauguration shown in [Table 2.16](#).

| Age at inauguration | Frequency |
|---------------------|-----------|
| 41.5–46.5 | 4 |
| 46.5–51.5 | 11 |
| 51.5–56.5 | 14 |
| 56.5–61.5 | 9 |
| 61.5–66.5 | 4 |
| 66.5–71.5 | 2 |

Table 2.16

Frequency polygons are useful for comparing distributions. This is achieved by overlaying the frequency polygons drawn for different data sets.

EXAMPLE 2.12

We will construct an overlay frequency polygon comparing the scores from [Example 2.11](#) with the students’ final numeric grade.

| Frequency distribution for calculus final test scores | | | |
|---|-------------|-----------|----------------------|
| Lower bound | Upper bound | Frequency | Cumulative frequency |
| 49.5 | 59.5 | 5 | 5 |
| 59.5 | 69.5 | 10 | 15 |
| 69.5 | 79.5 | 30 | 45 |
| 79.5 | 89.5 | 40 | 85 |
| 89.5 | 99.5 | 15 | 100 |

Table 2.17

| Frequency distribution for calculus final grades | | | |
|--|-------------|-----------|----------------------|
| Lower bound | Upper bound | Frequency | Cumulative frequency |
| 49.5 | 59.5 | 10 | 10 |
| 59.5 | 69.5 | 10 | 20 |
| 69.5 | 79.5 | 30 | 50 |
| 79.5 | 89.5 | 45 | 95 |
| 89.5 | 99.5 | 5 | 100 |

Table 2.18

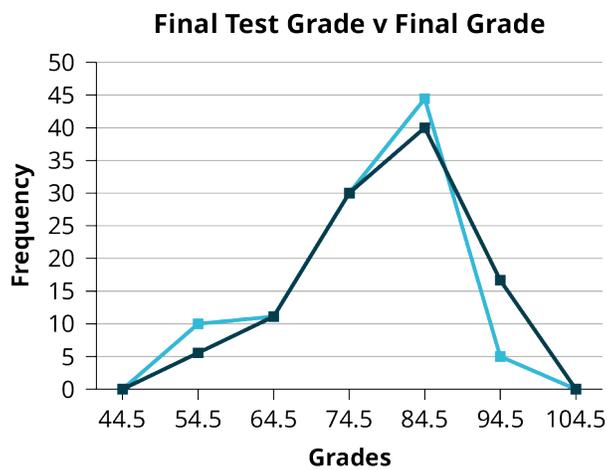


Figure 2.9

> TRY IT 2.12

We will construct an overlay frequency polygon comparing the scores from [Example 2.12](#) with the students' final test

scores in algebra.

| Frequency Distribution for Algebra Final Test Scores | | | |
|--|-------------|-----------|----------------------|
| Lower Bound | Upper Bound | Frequency | Cumulative Frequency |
| 49.5 | 59.5 | 10 | 10 |
| 59.5 | 69.5 | 5 | 15 |
| 69.5 | 79.5 | 40 | 55 |
| 79.5 | 89.5 | 35 | 90 |
| 89.5 | 99.5 | 10 | 100 |

Table 2.19

Constructing a Time Series Graph

Suppose that we want to study the temperature range of a region for an entire month. Every day at noon we note the temperature and write this down in a log. A variety of statistical studies could be done with these data. We could find the mean or the median temperature for the month. We could construct a histogram displaying the number of days that temperatures reach a certain range of values. However, all of these methods ignore a portion of the data that we have collected.

One feature of the data that we may want to consider is that of time. Since each date is paired with the temperature reading for the day, we don't have to think of the data as being random. We can instead use the times given to impose a chronological order on the data. A graph that recognizes this ordering and displays the changing temperature as the month progresses is called a time series graph.

To construct a time series graph, we must look at both pieces of our **paired data set**. We start with a standard Cartesian coordinate system. The horizontal axis is used to plot the date or time increments, and the vertical axis is used to plot the values of the variable that we are measuring. By doing this, we make each point on the graph correspond to a date and a measured quantity. The points on the graph are typically connected by straight lines in the order in which they occur.

EXAMPLE 2.13

Problem

The following data shows the Annual Consumer Price Index, each month, for ten years. Construct a time series graph for the Annual Consumer Price Index data only.

| Year | Jan | Feb | Mar | Apr | May | Jun | Jul |
|------|-------|-------|-------|-------|-------|-------|-------|
| 1 | 181.7 | 183.1 | 184.2 | 183.8 | 183.5 | 183.7 | 183.9 |
| 2 | 185.2 | 186.2 | 187.4 | 188.0 | 189.1 | 189.7 | 189.4 |
| 3 | 190.7 | 191.8 | 193.3 | 194.6 | 194.4 | 194.5 | 195.4 |
| 4 | 198.3 | 198.7 | 199.8 | 201.5 | 202.5 | 202.9 | 203.5 |

Table 2.20

| Year | Jan | Feb | Mar | Apr | May | Jun | Jul |
|------|---------|---------|---------|---------|---------|---------|---------|
| 5 | 202.416 | 203.499 | 205.352 | 206.686 | 207.949 | 208.352 | 208.299 |
| 6 | 211.080 | 211.693 | 213.528 | 214.823 | 216.632 | 218.815 | 219.964 |
| 7 | 211.143 | 212.193 | 212.709 | 213.240 | 213.856 | 215.693 | 215.351 |
| 8 | 216.687 | 216.741 | 217.631 | 218.009 | 218.178 | 217.965 | 218.011 |
| 9 | 220.223 | 221.309 | 223.467 | 224.906 | 225.964 | 225.722 | 225.922 |
| 10 | 226.665 | 227.663 | 229.392 | 230.085 | 229.815 | 229.478 | 229.104 |

Table 2.20

| Year | Aug | Sep | Oct | Nov | Dec | Annual |
|------|---------|---------|---------|---------|---------|---------|
| 1 | 184.6 | 185.2 | 185.0 | 184.5 | 184.3 | 184.0 |
| 2 | 189.5 | 189.9 | 190.9 | 191.0 | 190.3 | 188.9 |
| 3 | 196.4 | 198.8 | 199.2 | 197.6 | 196.8 | 195.3 |
| 4 | 203.9 | 202.9 | 201.8 | 201.5 | 201.8 | 201.6 |
| 5 | 207.917 | 208.490 | 208.936 | 210.177 | 210.036 | 207.342 |
| 6 | 219.086 | 218.783 | 216.573 | 212.425 | 210.228 | 215.303 |
| 7 | 215.834 | 215.969 | 216.177 | 216.330 | 215.949 | 214.537 |
| 8 | 218.312 | 218.439 | 218.711 | 218.803 | 219.179 | 218.056 |
| 9 | 226.545 | 226.889 | 226.421 | 226.230 | 225.672 | 224.939 |
| 10 | 230.379 | 231.407 | 231.317 | 230.221 | 229.601 | 229.594 |

Table 2.21

✓ Solution

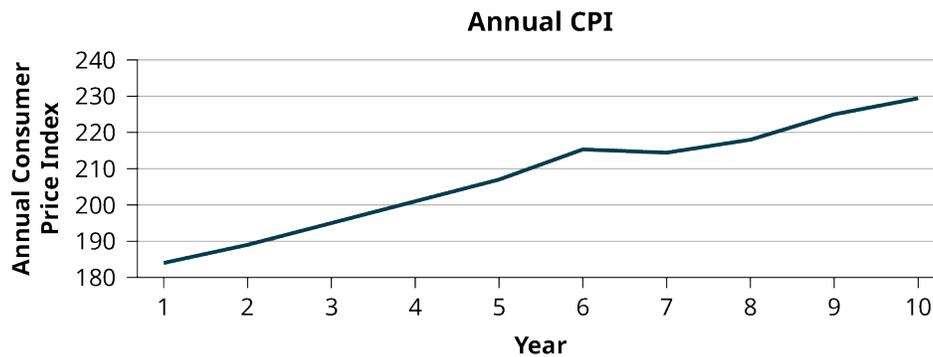


Figure 2.10

> TRY IT 2.13

The following table is a portion of a data set from www.worldbank.org. Use the table to construct a time series graph for CO₂ emissions for the United States.

| CO ₂ emissions | | | |
|---------------------------|---------|----------------|---------------|
| Year | Ukraine | United Kingdom | United States |
| 1 | 352,259 | 540,640 | 5,681,664 |
| 2 | 343,121 | 540,409 | 5,790,761 |
| 3 | 339,029 | 541,990 | 5,826,394 |
| 4 | 327,797 | 542,045 | 5,737,615 |
| 5 | 328,357 | 528,631 | 5,828,697 |
| 6 | 323,657 | 522,247 | 5,656,839 |
| 7 | 272,176 | 474,579 | 5,299,563 |

Table 2.22

Uses of a Time Series Graph

Time series graphs are important tools in various applications of statistics. When recording values of the same variable over an extended period of time, sometimes it is difficult to discern any trend or pattern. However, once the same data points are displayed graphically, some features jump out. Time series graphs make trends easy to spot.

How NOT to Lie with Statistics

It is important to remember that the very reason we develop a variety of methods to present data is to develop insights into the subject of what the observations represent. We want to get a "sense" of the data. Are the observations all very much alike or are they spread across a wide range of values, are they bunched at one end of the spectrum or are they distributed evenly and so on. We are trying to get a visual picture of the numerical data. Shortly we will develop formal mathematical measures of the data, but our visual graphical presentation can say much. It can, unfortunately, also say much that is distracting, confusing and simply wrong in terms of the impression the visual leaves. Many years ago Darrell Huff wrote the book *How to Lie with Statistics*. It has been through 25 plus printings and sold more than one and one-half million copies. His perspective was a harsh one and used many actual examples that were designed to mislead. He wanted to make people aware of such deception, but perhaps more importantly to educate so that others do not make the same errors inadvertently.

Again, the goal is to enlighten with visuals that tell the story of the data. Pie charts have a number of common problems when used to convey the message of the data. Too many pieces of the pie overwhelm the reader. More than perhaps five or six categories ought to give an idea of the relative importance of each piece. This is after all the goal of a pie chart, what subset matters most relative to the others. If there are more components than this then perhaps an alternative approach would be better or perhaps some can be consolidated into an "other" category. Pie charts cannot show changes over time, although we see this attempted all too often. In federal, state, and city finance documents pie charts are often presented to show the components of revenue available to the governing body for appropriation: income tax, sales tax motor vehicle taxes and so on. In and of itself this is interesting information and can be nicely done with a pie chart. The error occurs when two years are set side-by-side. Because the total revenues change year to year, but the size of the pie is fixed, no real information is provided and the relative size of each piece of the pie cannot be meaningfully compared.

Histograms can be very helpful in understanding the data. Properly presented, they can be a quick visual way to present probabilities of different categories by the simple visual of comparing relative areas in each category. Here the error, purposeful or not, is to vary the width of the categories. This of course makes comparison to the other categories impossible. It does embellish the importance of the category with the expanded width because it has a greater area, inappropriately, and thus visually "says" that that category has a higher probability of occurrence.

Time series graphs perhaps are the most abused. A plot of some variable across time should never be presented on axes that change part way across the page either in the vertical or horizontal dimension. Perhaps the time frame is changed from years to months. Perhaps this is to save space or because monthly data was not available for early years. In either case this confounds the presentation and destroys any value of the graph. If this is not done to purposefully confuse the reader, then it certainly is either lazy or sloppy work.

Changing the units of measurement of the axis can smooth out a drop or accentuate one. If you want to show large changes, then measure the variable in small units, penny rather than thousands of dollars. And of course to continue the fraud, be sure that the axis does not begin at zero, zero. If it begins at zero, zero, then it becomes apparent that the axis has been manipulated.

Perhaps you have a client that is concerned with the volatility of the portfolio you manage. An easy way to present the data is to use long time periods on the time series graph. Use months or better, quarters rather than daily or weekly data. If that doesn't get the volatility down then spread the time axis relative to the rate of return or portfolio valuation axis. If you want to show "quick" dramatic growth, then shrink the time axis. Any positive growth will show visually "high" growth rates. Do note that if the growth is negative then this trick will show the portfolio is collapsing at a dramatic rate.

Again, the goal of descriptive statistics is to convey meaningful visuals that tell the story of the data. Purposeful manipulation is fraud and unethical at the worst, but even at its best, making these type of errors will lead to confusion on the part of the analysis.

2.2 Measures of the Location of the Data

The common measures of location are **quartiles** and **percentiles**.

Quartiles divide an ordered data set into four equal parts. The three quartiles of a data set are labeled as Q1, Q2, and Q3.